The Optimal Arc Angles for Shooting a Basketball

There is an optimal arc angle that the shooter can send the ball to the center of the hoop with a minimum required amount of speed. The value for this arc angle depends on the shooting height of the player and his distance from the hoop.

By K. Frank Lin, Ph.D.
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K. Frank Lin *is an adjunct associate professor at the University of Waterloo, Ontario, Canada. He is also President of Lighting Sciences Canada Ltd. He has been a recreational basketball player for over 35 years. He stands 5 feet 5 inches, but he is well known and appreciated for his skyhooks by his fellow players.*

Introduction:

Since Dr. James Naismith invented the basketball game in 1891, many players have asked the question, "What is the optimal arc angle for shooting a basketball?" Many intelligent people would have guessed that the optimal angle is 45°. Based on the fact that the ball would have a larger hole to fall into if the ball approached the hoop more vertically, some people would say it is better to shoot with a high arc. Most good shooters do shoot the ball with a high arc angle. But how high is high enough? As a recreational player for over 35 years, I will try to find the answer to this question using physics knowledge, mathematics, and computer simulation. For purposes of my analysis, air resistance to the movement of the ball will be considered negligible.

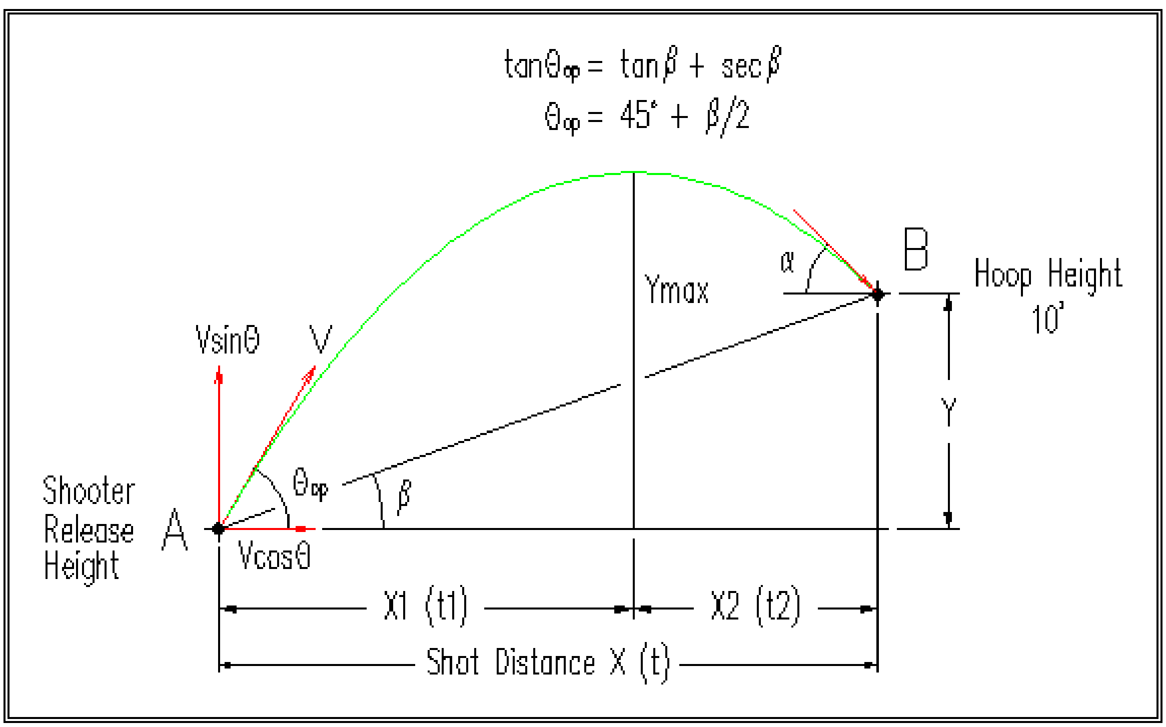

Figure 1

A Simplified Case: The ball is released at the same height as the hoop.

Consider the case when the ball is released at the same height as the hoop (Figure 1 with Y = 0). When the ball is released, the trajectory of the basketball is a parabola. If the hoop center is X ft. away when the ball is released, then the shot speed V and the upward arc angle θ must be related by equation 1.

$$X = HorizontalSpeed \cdot 2t_1 = \frac{VCos\theta \cdot 2VSin\theta}{G} = \frac{2V^2Cos\theta Sin\theta}{G}$$
$$= V^2 Sin2\theta / G \qquad ...1$$

Where G is 9.8 m/sec^2, and t_1 is the time for the ball to reach the maximum height.

The minimum speed, V_{min}, that can send the ball to the hoop center is equal to $\sqrt{GX}$, which is obtained by setting θ = 45°. When V is greater than V_{min}, two θ values are obtained for each V value.

For the purpose of finding the optimal arc angle, we can consider the ball is shot in the vertical plane that passes through the player and the hoop center.

Using a computer program (nicknamed Isaac), the combinations of θ and V that would allow the bottom of the ball to land at a designated position can be plotted out as a curve on a graph with the shot speed V as the horizontal axis and the angle θ as the vertical axis. Figure 2 was produced with the values of X set equal to 13.25 feet, 14 feet, and 14.75 feet.

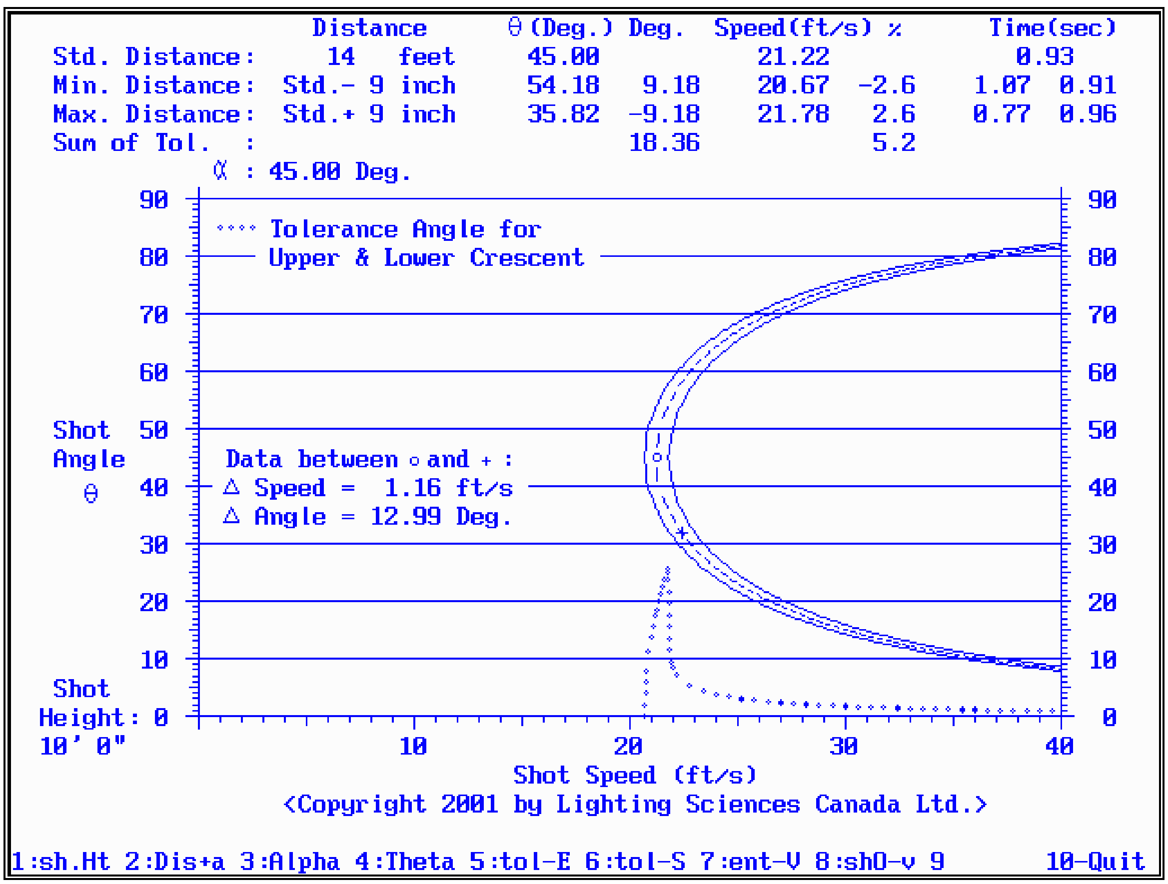

Figure 2

In figure 2, the right, center, and left curves are for the ball to land at the front rim, the hoop center, and the back rim, respectively. The crescent area between the left and the right curves indicates the acceptable combinations of θ and V that would allow the ball to land between the front and back of the rim, which would be considered a good shot.

Players never have perfect control over the speed and angle that they shoot the ball. To have a high percentage of good shots, certainly it is wise to shoot the ball in the zone where the acceptance area is the largest. From figure 2, the largest acceptance area is located at the front end of the crescent, with θ equal to 45°. If X is set to different values (but not too small), similar curves are produced by Isaac.

Hence, we have reached a conclusion. When the ball is released at the same height as the hoop, the optimal arc angles for shooting a basketball is 45°. At this angle, the speed that sends the ball to the center is the minimum speed required by the ball to reach the hoop center at any angle.

General Case: The ball is released Y feet lower than the hoop.

In most situations the hoop is Y ft. higher than the release height of the ball (Figure 1). Taking a clue from the simplified case, let us find the minimum speed and the associated arc angle to send the ball from A to B.

The trajectory of the basketball can be defined by the following set of equations:

$$\begin{cases} VSin\theta = Gt_1 & \ldots 2 \quad (The \quad ball \quad is \quad at \quad H_{\max}.) \\ X = VCos\theta \cdot (t_1 + t_2) & \ldots 3 \\ Y = \frac{G}{2}\left(t_1^2 - t_2^2\right) & \ldots 4 \end{cases}$$

From equation 4 we can obtain the following:

$$Y = \frac{G}{2}(t_1 + t_2)(t_1 - t_2) = \frac{G}{2}(t_1 + t_2)[2t_1 - (t_1 + t_2)]$$

Substituting in equations 2 and 3 we get:

$$\begin{aligned} Y &= \frac{G}{2} \cdot \frac{X}{VCos\theta} \cdot \left(\frac{2VSin\theta}{G} - \frac{X}{VCos\theta} \right) \\ &= XTan\theta - \frac{G}{2} \cdot \frac{X^2}{V^2 Cos^2\theta} \\ &= XTan\theta - \frac{GX^2}{2V^2}\left(Tan^2\theta + 1\right) \end{aligned}$$

$$GX^2 Tan^2\theta - 2V^2 XTan\theta + \left(2YV^2 + GX^2\right) = 0 \qquad \ldots 5$$

If we take the quadratic root of equation 5 we obtain:

$$Tan\theta = \frac{V^2 \pm \sqrt{V^4 - 2GYV^2 - G^2 X^2}}{GX} \qquad \ldots 6$$

You will note that as in the special case, equation 6 indicates that there are generally two θ values for each V value.

When $V^4 - 2GYV^2 - G^2X^2 < 0$ in equation 6, there is no possible angle for the ball to reach B. When $V^4 - 2GYV^2 - G^2X^2 = 0$ then $V^2 = V_{op}^2$ and $\theta = \theta_{op}$. Since $V_{op}^2 > 0$, we obtain the following:

$$V_{op}^2 = G\left(Y + \sqrt{Y^2 + X^2}\right) \qquad ...7$$

$$Tan\theta_{op} = \frac{V_{op}^2}{GX} = \frac{Y + \sqrt{X^2 + Y^2}}{X} \qquad ...8$$

$$Tan\theta_{op} = Tan\beta + Sec\beta \qquad \left(Tan\beta = \frac{Y}{X}\right)$$

$$Tan\beta + Sec\beta = \frac{1 + Sin\beta}{Cos\beta} = \frac{1 - Cos(90° + \beta)}{Sin(90° + \beta)} = Tan\left(\frac{90° + \beta}{2}\right)$$

$$\boxed{\Rightarrow \theta_{op} = 45° + \frac{\beta}{2}}$$

V_{op} is the minimum speed required to send the ball from A to B, and there is one θ_{op} value for each V_{op} value.

Using Isaac, the combinations of θ and V can again be plotted and the curves generated as in the special case. In figure 3, X has again been set to 13.25 feet, 14 feet, and 14.75 feet and the player shot height to 7 feet.

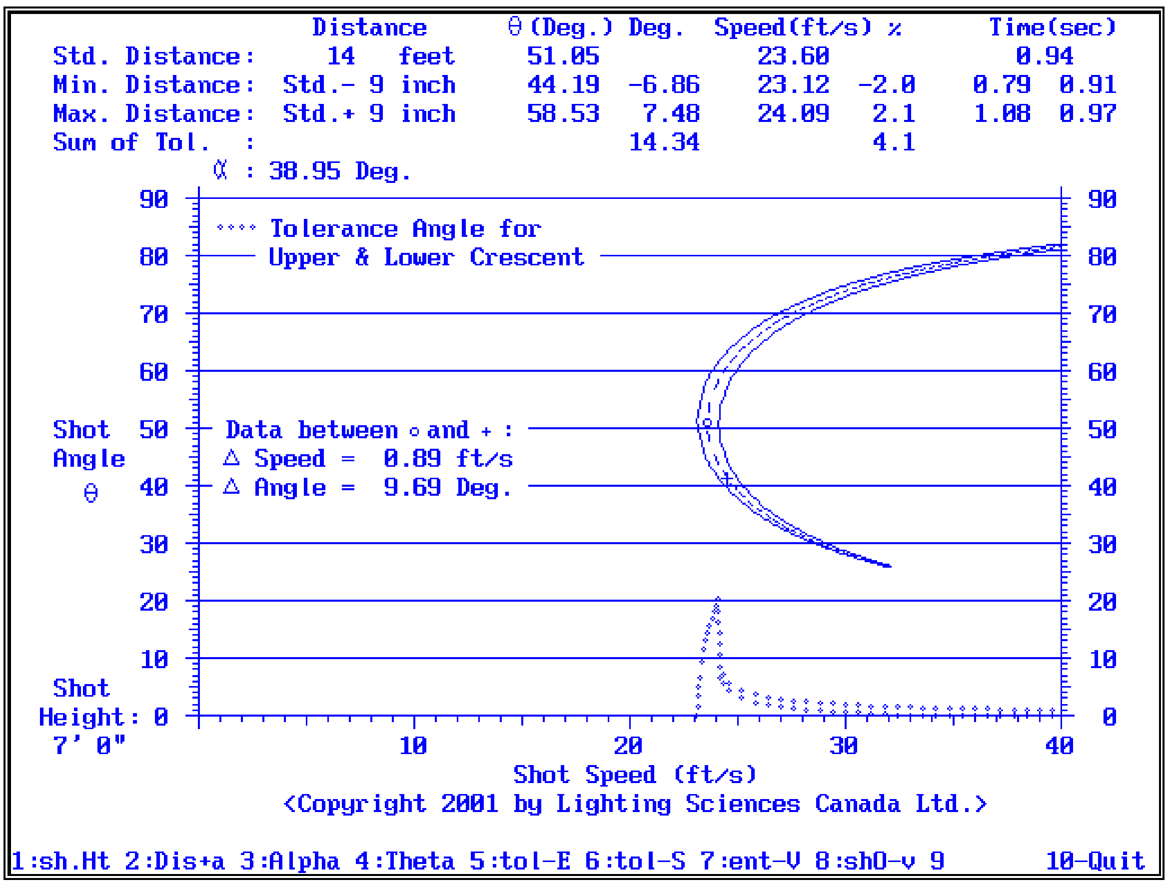

Figure 3

In figure 3, the largest acceptance area is again located at the front end of the crescent. Notice that the crescent has shifted up but retains the same general shape as the special case. Also, the largest acceptance area is located about the point where $\theta = \theta_{op}$ and $V = V_{op}$.

Hence we can conclude that, for a vertical offset between the player shot height and the hoop of Y, the optimal arc angle for shooting the basketball is θ_{op}. At this angle, the speed that sends the ball to the center of the hoop is V_{op}.

The valid zone (the area inside the crescent) in figure 3 is visually decreased from that of figure 2. It is obvious that higher shooting heights have advantage over lower shooting heights.

Now let's look at the underhand "granny" shot. The shot distance is reduced by one foot for an obvious reason. A four-foot shooting height is a good estimation. In figure 5, the valid zone is further reduced from figure 4. Due to low shooting height, the granny shot actually has less chance of getting into the hoop. This is contrary to the statement in an article by Curtis Rist in Discover magazine (August 7, 2008).

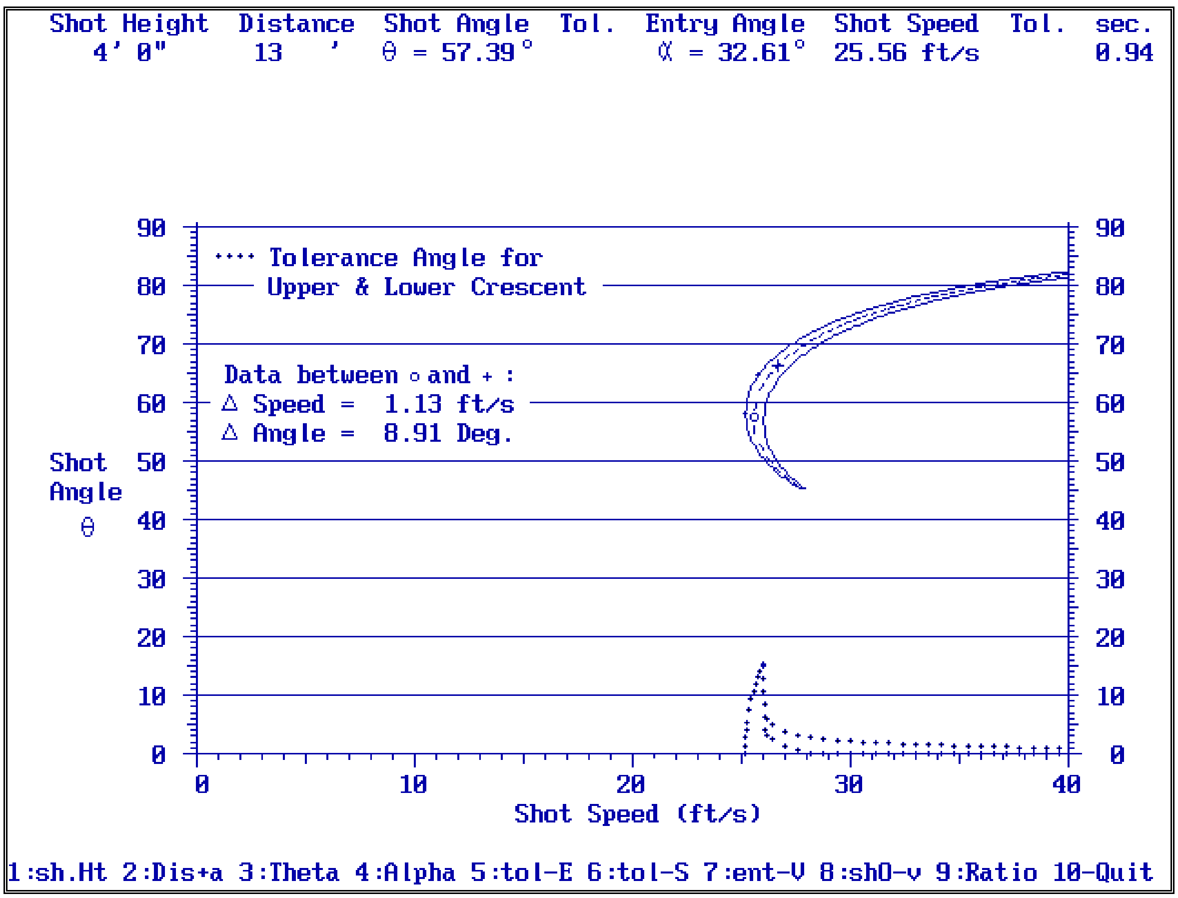

Figure 5

A table of θ_{op} and V_{op} values for different shot release heights versus a standard hoop height of 10 feet can be calculated and is presented below. You will note that the optimal angles are all greater than 45°.

Table 1

Dr. Frank Lin' s Optimal Arc Angles for Shooting the Basketball

Shot Height		Shot Distance					
		9’	11’	14’	17’	20’	24’
3’ 6”	Shot Angle (Deg.)	62.92	60.29	57.45	55.46	54.00	52.58
	Shot Speed (ft/s)	23.79	24.90	26.56	28.18	29.75	31.76
	Flight Time (sec.)	.83	.89	.98	1.06	1.14	1.24
4’	Shot Angle (Deg.)	61.85	59.31	56.60	54.72	53.35	52.02
	Shot Speed (ft/s)	23.25	24.41	26.13	27.79	29.40	31.44
	Flight Time (sec.)	.82	.88	.97	1.06	1.14	1.24
4’ 6”	Shot Angle (Deg.)	60.71	58.28	55.72	53.96	52.69	51.45
	Shot Speed (ft/s)	22.71	23.92	25.70	27.41	29.05	31.12
	Flight Time (sec.)	.81	.87	.97	1.05	1.14	1.24
5”	Shot Angle (Deg.)	59.53	57.22	54.83	53.19	52.02	50.88
	Shot Speed (ft/s)	22.18	23.44	25.27	27.03	28.70	30.81
	Flight Time (sec.)	.80	.87	.96	1.05	1.13	1.23
5’ 5”	Shot Angle (Deg.)	58.28	56.12	53.91	52.41	51.34	50.31
	Shot Speed (ft/s)	21.64	22.95	24.85	26.65	28.35	30.49
	Flight Time (sec.)	.79	.86	.96	1.05	1.13	1.23
6’	Shot Angle (Deg.)	56.98	54.99	52.97	51.62	50.65	49.73
	Shot Speed (ft/s)	21.10	22.47	24.43	26.27	28.01	30.18
	Flight Time (sec.)	.78	.85	.95	1.04	1.13	1.23
6’ 6”	Shot Angle (Deg.)	55.63	53.83	52.02	50.82	49.96	49.15
	Shot Speed (ft/s)	20.57	21.99	24.01	25.90	27.66	29.87
	Flight Time (sec.)	.78	.85	.95	1.04	1.12	1.23
7’	Shot Angle (Deg.)	54.22	52.63	51.05	50.00	49.27	48.56
	Shot Speed (ft/s)	20.04	21.52	23.60	25.52	27.33	29.57
	Flight Time (sec.)	.77	.84	.94	1.04	1.12	1.23
7’ 6”	Shot Angle (Deg.)	52.76	51.40	50.06	49.18	48.56	47.97
	Shot Speed (ft/s)	19.51	21.05	23.19	25.16	26.99	29.26
	Flight Time (sec.)	.76	.84	.94	1.03	1.12	1.23
8’	Shot Angle (Deg.)	51.26	50.15	49.07	48.35	47.86	47.38
	Shot Speed (ft/s)	18.99	20.59	22.78	24.79	26.66	28.96
	Flight Time (sec.)	.76	.83	.94	1.03	1.12	1.22
8’ 6”	Shot Angle (Deg.)	49.73	48.88	48.06	47.52	47.14	46.79
	Shot Speed (ft/s)	18.48	20.13	22.38	24.43	26.33	28.66
	Flight Time (sec.)	.75	.83	.94	1.03	1.12	1.22
9’	Shot Angle (Deg.)	48.17	47.60	47.04	46.68	46.43	46.19
	Shot Speed (ft/s)	17.98	19.68	21.99	24.08	26.00	28.36
	Flight Time (sec.)	.75	.83	.93	1.03	1.12	1.22
9’ 6”	Shot Angle (Deg.)	46.59	46.30	46.02	45.84	45.72	45.60
	Shot Speed (ft/s)	17.49	19.24	21.60	23.73	25.68	28.07
	Flight Time (sec.)	.75	.83	.93	1.03	1.12	1.22
10’	Shot Angle (Deg.)	45.00	45.00	45.00	45.00	45.00	45.00
	Shot Speed (ft/s)	17.01	18.81	21.22	23.38	25.36	27.78
	Flight Time (sec.)	.75	.83	.93	1.03	1.12	1.22

CCD Camera Measuring System:

Using recorded video images of a basketball shot, the shot angles and shot speeds can be calculated. If modern CCD cameras and machine vision technologies are used, the shot speeds and the shot angles can be calculated in less than 10 seconds. Comparing them to the optimal shot angles and shot speeds, the player can attempt necessary adjustments for his or her next shot. The shooting drill will be more efficient. The player can improve faster or even reach a shooting percentage never possible before.

A calibrated CCD camera system (nicknamed ArcMaster) has been used to record the trajectory of the basketball as it flies through the air. The set-up is shown in figure 4. The camera head was located 14 feet away from the player-hoop plane. The camera head was on the same level as the hoop, which typically is 10 feet high.

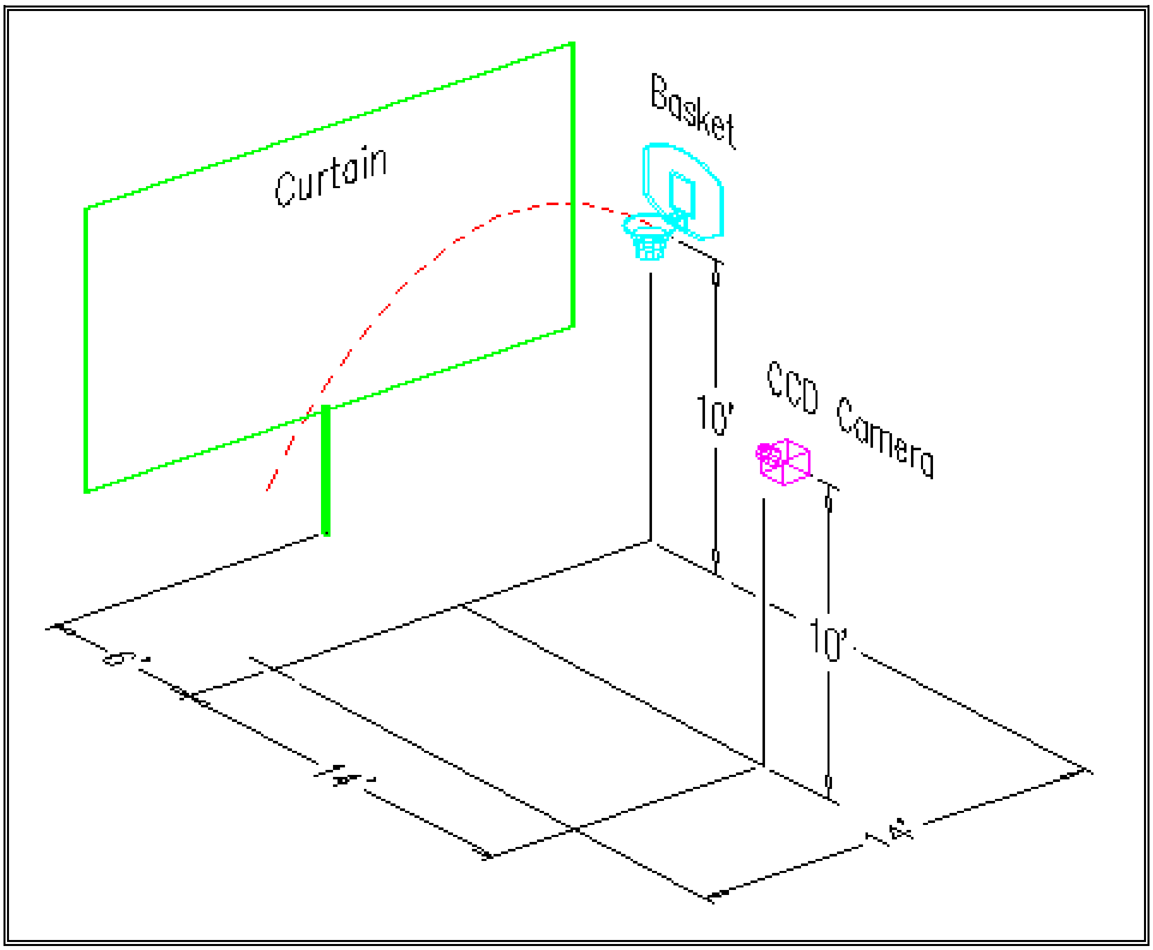

Figure 4

The player would get ready for a shot when he heard a two-click audio signal. He would then have to make a shot within one second after he heard a third click. The camera would then be record for two and a half seconds after this click. To increase contrast, a black curtain was placed in a plane six feet behind the player-hoop plane. A recorded image is shown in figure 5. Also shown are the best-fitted trajectory and the calculated shot speed and shot angle.

ArcMaster can keep records of shot angles and shot speeds along with the shooter's name. This makes possible future study of the statistics and trends of shooters.

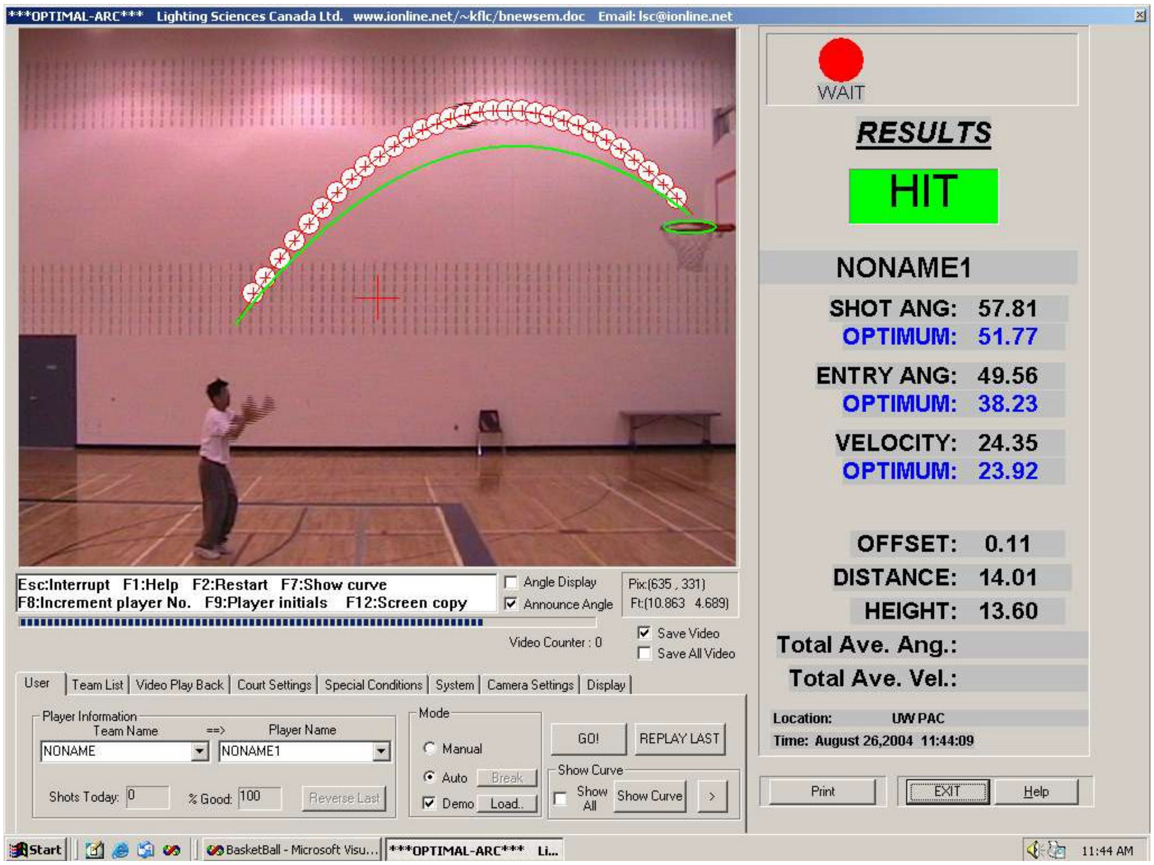

Figure 5

Conclusion:

At optimal arc angles, the ball can travel to the hoop center with minimum required speed. Many good shooters are frequently described as having a soft touch on the ball. This is because they are already shooting around the optimal arc angles and optimal speeds.

No doubt, the discussion on the optimal arc angles to shoot a basketball will continue for some time. Based on the fact that overshoots may hit the backboard and drop into the hoop and that it may create more favorable rebounding situations for the offense, some players will continue to shoot higher than optimal arc angle. Three or four degrees higher than the optimal arc angle may be the advisable limit. Otherwise, practice around the optimal arc angle and enjoy the game.